King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Information and Computer Science Department

ICS 253: Discrete Structures

Spring Semester 2021-2022 (212)
Midterm Exam, Tuesday 15 March 2022
Duration: 90 Minutes – Section «Section»

«First_Name» «Last_Name» **«Seat»**

«Uername» **«Serial»**

This Exam has 2 parts. Part I has 3 problem solving questions worth 25 points and Part II has 25 Multiple-choice questions (MCQ) worth 75 points.

Important Instructions:

1. The exam is closed book and closed notes.
2. All types of calculators, pagers or mobile phones are NOT allowed during the examination.
3. Answer all questions and make sure your answers are **clear** and **readable** and only in the indicated places clearly. The last page is a reference sheet that you may tear it off.

Part I. [25 Points] Problem Solving Questions

Question 1: (5 points) Find

$$\sum_{k=100}^{499} k$$

Show the steps used in your solution. Note: No need to compute the final answer (e.g. if the correct answer is $195 \times \frac{782}{2}$, you may leave it as such.)

$$\sum_{k=100}^{499} k = \sum_{k=1}^{499} k - \sum_{k=1}^{99} k = \frac{499(500)}{2} - \frac{99(100)}{2} = 499(250) - 99(50)$$

OR

Let $j = k - 99 \leftrightarrow k = j + 99$. Then,

$$\sum_{k=100}^{499} k = \sum_{j=1}^{400} (j + 99) = \sum_{j=1}^{400} j + \sum_{j=1}^{400} 99 = \frac{400(401)}{2} + 400(99) = 200(401) + 400(99)$$

Question 2: (10 points) Show that $(r \wedge s) \rightarrow (r \vee s)$ is a tautology. Use logical equivalences to demonstrate that it is logically equivalent to **T**. Show your steps and mention the rules you have used. Don't use a truth table.

$(r \wedge s) \rightarrow (r \vee s) \equiv \neg(r \wedge s) \vee (r \vee s)$ by Implication definition

$\equiv (\neg r \vee \neg s) \vee (r \vee s)$ by the first De Morgan law

$\equiv (\neg r \vee r) \vee (\neg s \vee s)$ by the associative and commutative laws for disjunction

$\equiv T \vee T$ by negation law and the commutative law for disjunction

$\equiv T$ by the domination law

Question 3: (10 points) Prove that $a < b$ if and only if $\frac{a+b}{2} > a$.

$$a < b \leftrightarrow a + a < a + b \leftrightarrow \frac{2a}{2} < \frac{a+b}{2} \leftrightarrow a < \frac{a+b}{2}$$

OR

$$a < b \rightarrow 2a < a + b \rightarrow \frac{2a}{2} < \frac{a+b}{2} \rightarrow a < \frac{a+b}{2}$$

AND

$$\frac{a+b}{2} > a \rightarrow a + b > 2a \rightarrow b > a$$

Part II. [75 points]: MCQ (3 points each)

Write X for your choice to each question after answering all MCQ questions (Grading will be based on the choices in this table only).

II		Your choice for each question				
		(a)	(b)	(c)	(d)	(e)
	Example			X		
Question Number	1					X
	2			X		
	3				X	
	4			X		
	5		X			
	6			X		
	7	X				
	8		X			
	9					X
	10				X	
	11					X
	12	X				
	13				X	
	14		X			
	15	X				
	16				X	
	17					X
	18			X		
	19	X				
	20		X			
	21		X			
	22					X
	23	X				
	24			X		
	25				X	

1) If a set S has 4 elements, then its power set $P(S)$ has ________ elements.

(a) 1 (b) 2 (c) 4 (d) 8 (e) 16

2) Consider the set $S = \{ (x, y) \mid x^2 + y^2 = z^2$, where x, y and z are all integers$\}$. Which of the following ordered pairs is not an element in S?

(a) (3, 4) (b) (12, 5) (c) (1, 1) (d) (0, 1) (e) (4, 3)

3) The statement $\forall x \forall y \forall z \exists w\big((w \neq x) \wedge (w \neq y) \wedge (w \neq z)\big)$ is true if all variables belong to the domain
   (a) $\{-1\}$
   (b) $\{0,1\}$
   (c) $\{\pi, e, \sqrt{2}\}$
   (d) $\{1,2,3,4,5\}$
   (e) None of the above answers is correct.

Let $Q(x,y)$ be the statement "Student x has participated in competition y.", where the domain for x consists of all students at your school and for y consists of all competitions. Answer Questions **4, 5** and **6** related to this statement.

4) The statement: "Every competition has had a student from your school as a participant." is correctly expressed in terms of $Q(x,y)$, quantifiers, and logical connectives as:
   (a) $\exists x \forall y\, Q(x,y)$
   (b) $\forall x \exists y\, Q(x,y)$
   (c) $\forall y \exists x\, Q(x,y)$
   (d) $\exists y \forall x\, Q(x,y)$
   (e) Both answers (a) and (c) are correct

5) The statement: "No student at your school has ever participated in any competition." is correctly expressed in terms of $Q(x,y)$, quantifiers, and logical connectives as:
   (a) $\neg\exists x \forall y\, Q(x,y)$
   (b) $\neg\exists x \exists y\, Q(x,y)$
   (c) $\neg\forall x \forall y\, Q(x,y)$
   (d) $\neg\exists x \neg\exists y\, Q(x,y)$
   (e) More than one answer above is correct

6) The statement: "There is exactly one student from your school who participated in the Hackathon competition." is correctly expressed in terms of $Q(x,y)$, quantifiers, and logical connectives as:
   (a) $\exists x \exists y\, Q(x,y)$
   (b) $\exists x\, Q(x, Hackathon)$
   (c) $\exists x \forall z \Big(Q(x, Hackathon) \wedge \big(Q(z, Hackathon) \rightarrow (z = x)\big)\Big)$
   (d) $\exists x \forall z \Big(Q(x, Hackathon) \wedge \big(Q(z, Hackathon) \leftrightarrow (z = x)\big)\Big)$
   (e) More than one answer above is correct.

7) Let Ai = {1, 2, 3, …, i}. Therefore, $\bigcup_{i=1}^{n} A_i =$
   (a) $\{1, 2, 3, \ldots, n\}$ (b) 1, 2, 3} (c) {1} (d) $\{n\}$ (e) ϕ

8) Suppose that x is not an integer. Therefore, $\lfloor x \rfloor + \lceil x \rceil = ?$
   (a) $2x$ (b) $2\lfloor x \rfloor + 1$ (c) $2x - 1$ (d) $2\lceil x \rceil + 1$ (e) $2x - 2$

9) Which of the following functions $f(x)$ is a bijection from **R** to **R**:
   (a) $\sqrt{x}$ (b) $x^2 + 1$ (c) $\lfloor x^3 \rfloor$ (d) $\dfrac{x^2+1}{x^2+2}$ (e) $2x + 1$

10) Assume that the domain of each variable below consists of all real numbers. Which of the following statements is true?
   (a) $\boldsymbol{\forall x \exists y (x^2 = y)}$.
   (b) $\exists x \forall y (xy = 0)$.
   (c) $\boldsymbol{\exists x \forall y (y \neq 0 \rightarrow xy = 1)}$.
   (d) Only (a) and (b) are true.
   (e) (a), (b) and (c) are all true.

11) For the sequence of integers 2, 6, 12, 20, 30, …, what is the next integer in the sequence:
   (a) 24 (b) 28 (c) 36 (d) 40 (e) 42

12) Find $\prod_{i=0}^{10} i$.

(a) 0 (b) 1 (c) 10! (d) 10^{10} (e) i^{10}

13) Let $A_i = \left\{\frac{1}{1}, \frac{1}{2}, \dots, \frac{1}{i}\right\}, i = 1,2,3 \dots.$ Then, $\bigcup_{i=1}^{\infty} A_i$ is

(a) undefined.
(b) ϕ
(c) finite.
(d) infinitely countable.
(e) uncountable.

14) Given the following premises: "Every student has an Internet account."
"Ali does not have an Internet account."
"Saleh has an Internet account."

We can conclude that
(a) Ali is a student.
(b) Ali is not a student.
(c) Saleh is a student.
(d) Saleh is not a student.
(e) Answers (b) and (c) are both concluded.

15) How many rows appear in a truth table for the compound proposition

$q \lor p \lor \neg s \lor \neg r \lor \neg t \lor u$?

(a) 64 (b) 32 (c) 16 (d) 128
(e) None of the other answers is correct.

16) Given the hypotheses: "Fouad is a bad boy or Noura is a good girl."
"Fouad is a good boy or Dawood is happy"

We can conclude that
(a) Dawood is happy.
(b) Noura is a good girl.
(c) Dawood is happy and Noura is a good girl.
(d) Dawood is happy or Noura is a good girl.
(e) None of the above can be concluded.

17) Given the following conditional statements:

1. If 1 + 1 = 2, then 2 + 2 = 5.
2. If 1 + 1 = 3, then 2 + 2 = 4
3. If 1 + 1 = 3, then 2 + 2 = 5
4. If cats can fly, then 1 + 1 = 3

Which conditional statement(s) is/are true?
(a) Only statements 1 is true.
(b) Only statements 2 is true.
(c) Only statement 3 is true.
(d) Only statements 2 and 3 are true.
(e) Only statements 2, 3, and 4 are true.

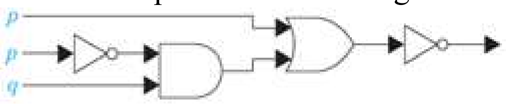
18) What is the contrapositive of the conditional statement: “If a positive integer is prime, then it has no divisors other than 1 and itself.”

(a) If a positive integer is not prime, then it has a divisor other than 1 and itself.
(b) If a positive integer has no divisors other than 1 and itself, then it is a prime.
(c) If a positive integer has a divisor other than 1 and itself, then it is not prime.
(d) If a positive integer is not prime, then it does not have a divisor other than 1 and itself.
(e) None of the other answers is correct.

19) Given the two bit-strings 00 0111 0001 and 10 0100 1000, find the bitwise *AND* and the bitwise *XOR* of the two bit-strings.

(a) Bitwise AND is 00 0100 0000; Bitwise XOR is 10 0011 1001.
(b) Bitwise AND is 10 0111 1001; Bitwise XOR is 10 0011 1001.
(c) Bitwise AND is 10 0111 1001; Bitwise XOR is 00 0100 0000.
(d) Bitwise AND is 00 0100 0000; Bitwise XOR is 10 0111 1001.
(e) None of the other answers is correct.

20) Find the output of the following combinatorial circuits. Select the correct answer.



(a) $\neg p \lor (\neg p \land q)$
(b) $\neg(p \lor (\neg p \land q))$
(c) $\neg(p \lor (p \land q))$
(d) $\neg(p \land (\neg p \lor q))$
(e) None of the other answers is correct.

21) Translate the following statement into propositional logic using the propositions provided.
“You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book”.
Express your answer in terms of:
g: “You can graduate,” *m*: “You owe money to the university,” *r*: “You have completed the requirements of your major,” and *b*: “You have an overdue library book.”

(a) $g \land (r \land (\neg m) \land (\neg b))$
(b) $g \rightarrow (r \land (\neg m) \land (\neg b))$
(c) $g \rightarrow (r \lor (\neg m) \lor (\neg b))$
(d) $g \lor (r \lor (\neg m) \lor (\neg b))$
(e) None of the other answers is correct

22) When three professors are seated in a restaurant, the host asks them: “Does everyone want coffee?” The first professor says: “I do not know.” The second professor then says: “I do not know.” Finally, the third professor says: “No, not everyone wants coffee.” The host comes back and gives coffee to the professors who want it. Assuming the host has understood the requests, how did he distribute the coffee?

(a) Brought coffee to all three professors.
(b) Brought coffee only to the first professor.
(c) Brought coffee only to the second professor.
(d) Brought coffee only to the first and the third professors.
(e) Brought coffee only to the first and the second professors.

23) Determine whether each of these compound propositions is satisfiable.

1. $(p \lor \neg q) \land (\neg p \lor q) \land (\neg p \lor \neg q)$
2. $(p \to q) \land (p \to \neg q) \land (\neg p \to q) \land (\neg p \to \neg q)$
3. $(p \leftrightarrow q) \land (\neg p \leftrightarrow q)$

(a) Only 1 is satisfiable.
(b) Only 2 is satisfiable.
(c) Only 3 is satisfiable.
(d) All the three compound propositions are satisfiable.
(e) None of the other answers is correct

24) Given that $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny", the domain consists of all people, and given the following two statements:

1. $\forall x(C(x) \to F(x))$
2. $\exists x(C(x) \land F(x))$

Choose the most suitable English Translation to the above two statements.

(a) Statement 1: Every comedian is funny; Statement 2: Some people are comedians.
(b) Statement 1: Every person is a funny comedian; Statement 2: Some comedians are funny.
(c) Statement 1: Every comedian is funny; Statement 2: Some comedians are funny.
(d) Statement 1: Every person is a funny comedian; Statement 2: There exists a person such that if he is a comedian, then he is funny.
(e) None of the other answers is correct.

25) Suppose that the domain of the propositional function $P(x)$ consists of the set $\{1, 2, 3, 4, 5\}$. Choose the most suitable answer that expresses the statement

$$\forall x\big((x \neq 3) \to P(x)\big) \lor \exists x \neg P(x)$$

without using quantifiers, instead using only negations, disjunctions, and conjunctions

(a) $(P(1) \lor P(2) \lor P(4) \lor P(5)) \lor (\neg P(1) \land \neg P(2) \land \neg P(3) \land \neg P(4) \land \neg P(5))$.
(b) $(P(1) \land P(2) \land P(3) \land P(4) \land P(5)) \lor (\neg P(1) \lor \neg P(2) \lor \neg P(4) \lor \neg P(5))$.
(c) $(P(1) \lor P(2) \lor P(3)\ ) \lor P(4) \lor P(5)) \lor (\neg P(1) \land \neg P(2) \land \neg P(4) \land \neg P(5))$.
(d) $(P(1) \land P(2) \land P(4) \land P(5)) \lor (\neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4) \lor \neg P(5))$.
(e) None of the above answers is correct

Formula Sheet

Logical Equivalences Table

Equivalence	Name
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws
$p \to q \equiv \neg p \vee q$	Conditional disjunction equivalence

TABLE 2 Some Useful Summation Formulae.

Sum	*Closed Form*
$\sum_{k=0}^{n} ar^k \ (r \neq 0)$	$\frac{ar^{n+1}-a}{r-1}, r \neq 1$
$\sum_{k=1}^{n} k$	$\frac{n(n+1)}{2}$
$\sum_{k=1}^{n} k^2$	$\frac{n(n+1)(2n+1)}{6}$
$\sum_{k=1}^{n} k^3$	$\frac{n^2(n+1)^2}{4}$
$\sum_{k=0}^{\infty} x^k, \lvert x\rvert < 1$	$\frac{1}{1-x}$
$\sum_{k=1}^{\infty} kx^{k-1}, \lvert x\rvert < 1$	$\frac{1}{(1-x)^2}$

$p \to (p \vee q)$	Addition	$[\neg q \wedge (p \to q)] \to \neg p$	Modus Tollens
$(p \wedge q) \to p$	Simplification	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$((p) \wedge (q)) \to (p \wedge q)$	Conjunction	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
$[p \wedge (p \to q)] \to q$	Modus Ponens	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

$A \cap U = A$ $A \cup \phi = A$	Identity Laws	$A \cup U = U$ $A \cap \phi = \phi$	Domination Laws
$A \cap A = A$ $A \cup A = A$	Idempotent Laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$\overline{(\bar{A})} = A$	Complementation Law	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$\overline{A \cap B} = \bar{A} \cup \bar{B}$ $\overline{A \cup B} = \bar{A} \cap \bar{B}$	De Morgan's Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws	$A \cup \bar{A} = U$ $A \cap \bar{A} = \phi$	Complement Laws

Some Useful Sequences	
n^2	1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, …
n^3	1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, …
n^4	1, 16, 81, 256, 625, 1296, 2401, 4096, 6561, 10000, 14641, …
2^n	2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, …
3^n	3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049, 177147, …
$n!$	1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800
f_n	1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … (Fibonacci)